value of all prizes. It is the total cash valueof all prizes because $E + c$ is the amount returned, averaged over all tickets. Therefore, we have the following:

$$\text{Expected Win on unit wager in a lottery} = \frac{\text{Total cash value of all prizes}}{\text{Total retail value of all tickets eligible for the draw}}$$

The marbles game can be thought of as a lottery where $E = 0.4$, $n = 5$ and $c = 2$. Therefore

$$\frac{n(E+c)}{nc} = \frac{5 \times (0.4+2)}{5 \times 2} = \frac{5 \times 2.4}{5 \times 2} = 1.2.$$

The Expected Win per unit cost is 1.2. The mathematical expectation per unit cost is 0.2.

A.6 St. Peterburg Paradox and resolution by Cramér

The game: A fair coin is flipped until it comes up heads the first time. At that point the player wins $\$2^n$, where n is the number of times the coin was flipped. How much should one be willing to pay for playing this game?

If the coin lands heads on the first flip you win \$2, if it lands heads on the second flip you win \$4, and if this happens on the third flip you win \$8, and so on. The probabilities of the outcomes are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$. So the mathematical expectation of the St. Petersburg game is

$$\frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \cdots = 1 + 1 + 1 + \cdots \longrightarrow \infty.$$

Using the square-root of the win, the marginal utility concept of Cramér, we get

$$\frac{1}{2} \times \sqrt{2} + \frac{1}{4} \times \sqrt{4} + \frac{1}{8} \times \sqrt{8} + \cdots \approx 2.9.$$