

C.2 Betting a large sum once versus betting \$1 many times in a lottery

Suppose there is a prize associated with chance p in a lottery and the tickets cost \$1. We consider one bet of n dollars in one draw of the lottery versus bets of \$1 each on n draws.

A Bernoulli trial is a random process with two outcomes and fixed chances for each. An example is the throw of a coin. A lottery can be viewed as a Bernoulli trial, for your single ticket either wins the jackpot or not. Or winning one of the prizes in the lotto, or not winning any prize. So playing \$1 n times constitutes n Bernoulli trials. But in n Bernoulli trials with probability p , the expected number of successes is np , regardless of the value of p . If a success means a positive payoff of an amount A , the mathematical expectation is npA .

(That the mathematical expectation is npA is seen also with a direct computation as follows. The probability for k successes in n Bernoulli trials with a single-trial probability of success p is

$$\binom{n}{k} p^k (1-p)^{n-k}.$$

Let $1-p = q$. In k successes, the payoff is kA . So mathematical expectation in the n trials is

$$\begin{aligned} \sum_{k=0}^n kA \times \binom{n}{k} p^k q^{n-k} &= \sum_{k=1}^n kA \times \binom{n}{k} p^k q^{n-k} = \\ npA \sum_{k=1}^n \binom{n-1}{k-1} p^{k-1} q^{n-k} &= npA \sum_{k=1}^n \binom{n-1}{k-1} p^{k-1} q^{(n-1)-(k-1)} = \\ npA \sum_{s=0}^{n-1} \binom{n-1}{s} p^s q^{(n-1)-s} &= npA(p+q)^{n-1} = npA, \end{aligned}$$

because $p+q=1$.)

On the other hand, if your n tickets are in a single lotto draw, a phenomenon called 'mutually exclusive events' comes into play.

With your n tickets in a draw, we consider two cases: (a) p is very small, as in the chance for a jackpot in a lotto, and (b) p is not very small, as in the case of the lower prizes in a lotto.

Case (a) is easy, for when p is very small and you have n tickets for a draw, your chance for a prize with the very small chance is np . The reason is that lotteries are based on the idea that prizes are won when numbers drawn by the lottery match all or some of the numbers on your ticket.