G

Mathematical support for Chapter 9

G.1 Chance of occurrence of a given number of jackpot winners in a lotto drawing

If r distinguishable marbles are randomly placed in n distinguishable urns, what is the chance $P(k,r,n)$ that an urn chosen at random will contain k of the marbles? This is the Classical Occupancy Problem (Chapter 2 in [16]) in Probability. The answer is

$$P(k,r,n) = \binom{r}{k} \frac{1}{n^k} \left(1 - \frac{1}{n}\right)^{r-k}. \tag{G.1}$$

The proof is immediate, for we can choose to label each marble in one of n ways so that there are n^r labelings, while for a specific urn to contain exactly $k (k = 0, 1, \ldots, r)$ marbles we choose k marbles in $\binom{r}{k}$ ways, and the remaining $r - k$ marbles into the remaining $n - 1$ urns in $(n-1)^{r-k}$ ways. Therefore

$$P(k,r,n) = \binom{r}{k} \cdot (n-1)^{r-k} \cdot \frac{1}{n^r} = \binom{r}{k} \frac{1}{n^k} \left(1 - \frac{1}{n}\right)^{r-k}.$$

The probability of the event of k jackpot winners in a lotto draw is an occupancy problem of this kind: r blocks are played in a draw in the lotto that has n blocks. $P(k,r,n)$ is the chance that the one block drawn by the lottery will be matched by each of k of the blocks played. The urn model applies because we can imagine the r blocks played as marbles.