Theorem 1. *In a pick-m lotto on n numbers, the chance for p of the m numbers chosen in a draw to be in a specific set of v of the n numbers is*

$$\frac{\binom{m}{p}\binom{n-m}{v-p}}{\binom{n}{v}}$$

which is also equal to

$$\frac{\binom{v}{p}\binom{n-v}{m-p}}{\binom{n}{m}}.$$

Proof. There are $\binom{n}{v}$ v-sets on n numbers. There are

$$\binom{m}{p}\binom{n-m}{v-p}$$

v-sets with p numbers from any given m-set. So the chance that a given v-set is one of the v-sets with p numbers from a given m-set is

$$\frac{\binom{m}{p}\binom{n-m}{v-p}}{\binom{n}{v}}.$$

On the other hand, there are $\binom{n}{m}$ m-sets on n numbers. There are

$$\binom{v}{p}\binom{n-v}{m-p}$$

m-sets with p numbers from any given v-set. So the chance that a given m-set is one of the m-sets with p numbers from a given v-set is

$$\frac{\binom{v}{p}\binom{n-v}{m-p}}{\binom{n}{m}}.$$

□