from Field 1 and one number from Field 2 whereas in Euromillions five numbers are picked from Field 1 and two numbers from Field 2. The lotto systems for pick-6 lotto we consider in this section are for both kinds of lotto games, for both use blocks of five numbers each from Field 1. To each block the player tacks on their favorite number or numbers from Field 2.

### 17.1.1 System with no pair of blocks sharing a number

A set of blocks sharing no numbers is a system with maximum chances - see Appendix M.6. Such a set (list) is like the one in Table 17.1.

| 1 | 2 | 3 | 4 | 5 |
| ---: | ---: | ---: | ---: | ---: |
| 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 |
| 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 |
| 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 |

Table 17.1. A list with 0 -overlap for pick-5 lotto
The list in Table 17.1 gives maximum chance for all prizes in pick- 5 lotto. It is not a wheel because it does not guarantee a win if the winning numbers are among the blocks. But it gives maximum chance for any win in a pick- 5 lotto on 45 or more numbers. We illustrate this in Appendix M. 6 for pick-6 lotto. Of course, the chance for the jackpot is the same as for any set with the same number of blocks.

You can also play fewer blocks from Table 17.1, or add more blocks if the field has more numbers. For example, if the lotto has 60 numbers in the field where you are required to pick the numbers from, then you can add the blocks $\{46,47,48,49,50\},\{51,52,53,54,55\}$, and $\{56,57,58,59,60\}$. To play the Powerball or Megamillions of the USA or the Euromillions, you can use these blocks and add on numbers from the second field of numbers in these games. Caution: Ensure to rearrange the numbers in the blocks if you play these systems - see Appendix J.2.

### 17.1.2 System with no pair of blocks sharing a pair of numbers

Maximum chances for lower prizes when using longer lists would have blocks sharing some numbers. In the best among them, no two
blocks share a pair of numbers. We give two systems for pick-5 lotto: one on 21 numbers, and another on 25 numbers.

The set of blocks in Table 17.2 is the Steiner System $S(2,5,21)$ on 21 numbers and block size 5 . Each pair of numbers from the 21 numbers occurs in exactly one block. So this is a least cost system with maximum chances. See Appendix M. 1 for its spectrum so you can compute the chances. While Steiner systems are beautiful, there are the most beautiful among them, comprising what is known in mathematics as Projective Planes. S( $2,5,21$ ) is a Projective Plane, in fact a special Projective Plane. $S(2,5,21)$ is a Projective Plane of order 4. It is like a geometry of just $4^{2}+4+1=21$ lines (blocks) on $4^{2}+4+1=21$ points (numbers), where each line (block) has $4+1=5$ points (numbers) and two points (numbers) are on a unique line (block). In other words, each pair of numbers is in some block and no pair of numbers appears in more than one block.

| $n=21, \quad k=5$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 2 | 3 | 4 | 5 | 12. | 3 | 9 | 17 | 20 | 21 |
| 2. | 1 | 6 | 9 | 11 | 19 | 13. | 3 | 10 | 11 | 14 | 16 |
| 3. | 1 | 7 | 13 | 16 | 17 | 14. | 4 | 6 | 7 | 10 | 20 |
| 4. | 1 | 8 | 10 | 18 | 21 | 15. | 4 | 8 | 11 | 15 | 17 |
| 5. | 1 | 12 | 14 | 15 | 20 | 16. | 4 | 9 | 12 | 16 | 18 |
|  | 2 | 6 | 14 | 17 | 18 | 17. | 4 | 13 | 14 | 19 | 21 |
|  | 2 | 7 | 11 | 12 | 21 | 18. | 5 | 6 | 15 | 16 | 21 |
| 8. | 2 | 8 | 16 | 19 | 20 | 19. | 5 | 7 | 8 | 9 | 14 |
| 9. | 2 | 9 | 10 | 13 | 15 | 20. | 5 | 10 | 12 | 17 | 19 |
| 10. | 3 | 6 | 8 | 12 | 13 | 21. | 5 | 11 | 13 | 18 |  |
|  | 3 | 7 | 15 | 18 | 19 |  |  |  |  |  |  |

Table 17.2. $\mathrm{S}(2,5,21)$
The set of blocks in Table 17.3 is the Steiner System $\mathrm{S}(2,5,25)$ on 25 numbers and block size 5 . Each pair of numbers from the 25 numbers occurs in exactly one block. So this is a least cost system with maximum chances. It is an example of a Steiner system that is not a projective Plane. See Appendix M. 2 for its spectrum so you can compute the chances.

| $n=25, \quad k=5$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 1 | 2 | 3 | 4 | 5 | 16. | 3 | 14 | 15 | 17 | 22 |
| 2. | 1 | 6 | 8 | 15 | 21 | 17. | 4 | 6 | 9 | 10 | 14 |
| 3. | 1 | 7 | 12 | 14 | 20 | 18. | 4 | 7 | 17 | 24 | 25 |
| 4. | 1 | 9 | 16 | 19 | 24 | 19. | 4 | 8 | 19 | 20 | 22 |
| 5. | 1 | 10 | 13 | 17 | 18 | 20. | 4 | 11 | 12 | 13 | 21 |
| 6. | 1 | 11 | 22 | 23 | 25 | 21. | 4 | 15 | 16 | 18 | 23 |
| 7. | 2 | 6 | 11 | 17 | 19 | 22. | 5 | 6 | 18 | 20 | 25 |

